Simple loss scaling laws for quadrupoles and higher-order multipoles used in antihydrogen traps

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Abstract. Simple scaling laws strongly suggest that for antihydrogen relevant parameters, quadrupole magnetic fields will transport particles into, or near to, the trap walls. Consequently quadrupoles are a poor choice for antihydrogen trapping. Higher order multipoles lead to much less transport.

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Recently, the ATHENA[1] and ATRAP[2] collaborations produced slow antihydrogen ($\bar{\text{H}}$) atoms at CERN. Both groups made $\bar{\text{H}}$ by storing its constituents, positrons (e^+) and anti-protons ($\bar{\text{p}}$), in adjacent sections of Penning-Malmberg traps, and then manipulating the trap potentials to mix the constituents.

The neutral $\bar{\text{H}}$ atoms were not confined by the charged-particle trapping fields, and quickly annihilated on the trap walls. While the production of cold $\bar{\text{H}}$ is remarkable, the most interesting proposed experiments, including tests of CPT invariance and gravitational interactions, require trapped $\bar{\text{H}}$. Because $\bar{\text{H}}$ cannot be easily moved to and then captured in an external trap, most trapping schemes entail superimposed neutral and charged-particle traps. The most commonly proposed schemes employ a magnetic multipole, typically a quadrupole, to make a minimum-B neutral trap. Since $\bar{\text{H}}$ is faintly diamagnetic, it will be attracted to the magnetic minimum. [3, 4].

Since the quadrupole fields destroy the cylindrical symmetry that underlies the Penning-Malmberg trap's outstanding performance[5], the $\bar{\text{H}}$ constituents might be lost before they form $\bar{\text{H}}$. Whether or not the particles will actually be lost has been fiercely debated, with papers arguing both that they will be lost[6, 7], that they would be confined[8, 9], and somewhere in between[10, 11, 12]. The experimental papers[6, 7, 9, 10, 11] were not definitive because they employed axial fields significantly lower than those used in the antihydrogen experiments, and the theory paper[8] discussed the behavior of single particles which is not obviously relevant for the dense particle clouds used in the experiments. Recently, however, we performed experiments[13] that show that confinement is quite poor in fields close to those used in the antihydrogen experiments. Many particles are lost immediately due to a hitherto unrecognized, but quite simple, single particle effect. This effect is alluded to in two

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recent publications[13, 14]; here we report the simple scalings that make this effect almost unavoidable for quadrupoles.

The total field in a quadrupole based trap is

$$\mathbf{B} = B_s\hat{z} + \beta_q(x\hat{x} - y\hat{y}). \tag{1}$$

where B_s is the solenoidal field and β_q gives the strength of the quadrupole field. (The complete field also includes mirror fields to confine the antihydrogen axially. We here ignore these fields as they do not qualitatively change the results.) The solenoidal field must be at least 1 T to properly confine and cool the charged particles. The trapping depth is proportional to the difference between the magnitude of the field at the wall and the magnitude of the field in the center:

$$\begin{aligned}\Delta B &= \sqrt{B_s^2 + (\beta_q R_w)^2} - B_s \\ &= B_s\left[\sqrt{1 + \left(\frac{\beta_q R_w}{B_s}\right)^2} - 1\right],\end{aligned}$$

where R_w is the trap wall radius. In the limit where the quadrupole field at the wall, $B_w = \beta_q R_w$, is small, this difference reduces to

$$\Delta B = \frac{1}{2}\left(\frac{\beta_q R_w}{B_s}\right)^2 B_s. \tag{2}$$

Antihydrogen is only weakly diamagnetic; a 1 T field increase produces a mere 0.67 K deep trap. The energy of the $\bar{\mathrm{H}}$ is relatively high[15, 16]. Unless methods are developed[17] to produce colder $\bar{\mathrm{H}}$, only a very small fraction of the $\bar{\mathrm{H}}$ will be trapped; for example, a 1 T well would capture less than one $\bar{\mathrm{H}}$ in 10^5 if the $\bar{\mathrm{H}}$ temperature is 2×10^3 K[15]. Taking 1 T as the minimum trapping depth, and remembering that the solenoidal field B_s is itself at least 1 T, requires that the ratio $\beta_q R_w / B_s$ must greater than unity; thus, the quadrupole field at the wall must be comparable to or greater than the solenoidal field.

Charged particles tend to follow magnetic field lines. Thus, a particle in the combined trap will follow the field lines given by Eq. 1. It is easy to show that there are field lines at $\pm\hat{x}$ that go exponentially outward as they progress in $\hat{z}$ [18]:

$$x(z) = x_0 \exp\left(\frac{\beta_q z}{B_s}\right), \tag{3}$$

where x_0 is the initial $\hat{x}$ position at $z = 0$. There are also field lines at $\pm\hat{y}$ that go exponentially inward as they progress in $\hat{z}$,

$$y(z) = y_0 \exp\left(-\frac{\beta_q z}{B_s}\right),. \tag{4}$$

However, it is also easy to show that the field lines that are not precisely on the axes converge towards the outwardly growing field lines[18].

The charged particle axial motion is limited by the electrostatic fields in the Penning-Malmberg trap. If a particle follows a field line into the trap wall before it is turned around by these electrostatic fields, it will be lost. As the particles on the $\pm\hat{x}$ axes progress outwards fastest, they will be lost most easily. The criterion that determines whether or not they are lost is

$$R_w \leq x_0 \exp\left(\frac{\beta_q \Delta z}{B_s}\right), \tag{5}$$

where Δz is the effective distance the particles travel.

Even if a particle is not on one of the $\pm\hat{x}$ axes, it will travel outwards almost as much because the other field lines converge towards the $\pm\hat{x}$ field lines. Furthermore, the particles rotate azimuthally from the magnetron fields at the electrostatic well ends and the particles self-field, and, thus, will quickly find themselves on the $\pm\hat{x}$ field lines. We can generalize the loss criterion Eq. 5 by substituting any radius initial r_0 (at $z = 0$) for x_0.

If particles enter into the quadrupole field from one side, Δz is the full length that they travel. If particles are initially in a pure solenoidal field, and the quadrupole field is then applied, Δz is half the length of the particle cloud. For the latter case, Eq. 5 becomes

$$R_w \leq r_0 \exp\left(\frac{1}{2}\frac{\beta_q L}{B_s}\right), \tag{6}$$

where L is the length of the particle cloud.

The electrostatic well in a Penning-Malmberg trap is formed from a series of stacked cylinders; at a minimum, three cylinders, biased $-$, $+$, and $-$, are required to make a well for negative particles. It is well known that this center cylinder cannot be made too short else the positive potential at the cylinder's wall will not penetrate into the cylinder's radial center. Figure 1 shows a graph of the potential at the center of a cylinder held at potential $\Phi = 1$, as a function of the cylinder's length S. Typically, the shortest cylinders used to confine particles are no shorter than $S = R_w$. For such short cylinders, the particles travel typically extends over close to the entire cylinder. Thus, the loss criterion becomes

$$R_w \leq r_0 \exp\left(\frac{1}{2}\frac{\beta_q R_w}{B_s}\right), \tag{7}$$

As discussed above, $\beta_q R_w / B_s$ must be at least one to make a functional well depth. If we set this ratio to unity, then the exponential in the criterion evaluates to $\exp(0.5) = 1.65$; particles whose initial radius is any greater than $0.61 R_w$ will be lost immediately. Realistically, the limit is much stricter for several reasons:

1. The ratio $\beta_q R_w / B_s$ is likely to be greater than one. For example, the ALPHA collaboration is constructing a multipole in which $\beta_q R_w / B_s \approx 1.7$[14]. For this strength field, particles at radius greater than $0.43 R_w$ will be lost.
2. For the constituent mixing scheme originally used by ATHENA and ATRAP to create antihydrogen, the travel length was much greater than $S = R_w$. For this type of mixing scheme, it would be hard to make the mixing length any shorter than

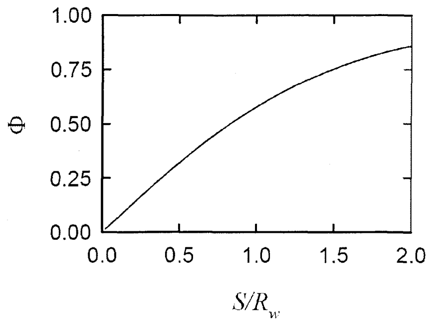

FIGURE 1. Potential in the center of an electrode of length S, biased to a potential of 1, and surrounded by infinite length, grounded cylinders.

$3R_w$. Moreover, the antiprotons are launched into the mixing region from the side, so Δz equals L, not $L/2$. Thus, at $\beta_q R_w/B_s = 1.7$, particles with radius greater than $0.05R_w$ will be lost.

3. Even if particles are not lost immediately, the loss separatrix is much closer to the particles after the multipole is applied. Much less radial expansion lead to loss.

Quadrupoles are not the only way to create a magnetic minimum. Higher order multipoles will also make a minimum. The field from a multipole scales as $(r/R_w)^{s-1}$, where s is 2 for a quadrupole, 3 for an sextupole, 4 for an octupole, etc. For the same field at the trap wall, higher order multipoles have lower fields near the trap center. Moreover, the field lines expand algebraically rather than exponentially. For typical parameters, the field line expansion is much lower as the multipole order is increased. The relation equivalent to Eq. 6 is

$$R_w \leq \frac{r_0}{\left[1 - \frac{(s-2)}{2}\frac{B_w}{B_s}\frac{r_0^{s-2}L}{R_w^{s-1}}\right]^{\frac{1}{s-2}}}, \tag{8}$$

where B_w is the field of the multipole at the trap wall. For example, for $B_w/B_s = 1.7$, $L/R_w = 2$, the field lines going through $r_0/R_w = 0.25$ at $z = 0$ will have expanded by a factor of 5.5 in a quadrupole, but only 1.1 in an octupole.

In conclusion, simple scaling laws show that particles will follow field lines into the trap walls for most antihydrogen relevant system configurations when a quadrupole is employed. Even if the the field lines do not hit the walls, they come sufficiently close that diffusive transport into the wall will be greatly enhanced. These results have been confirmed by PIC simulations[19], and agree with recent experimental measurements[13]. Consequently, the new apparatus constructed by the ALPHA collaboration uses an octupole magnet. The design of the magnet is described in Ref. [14].

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